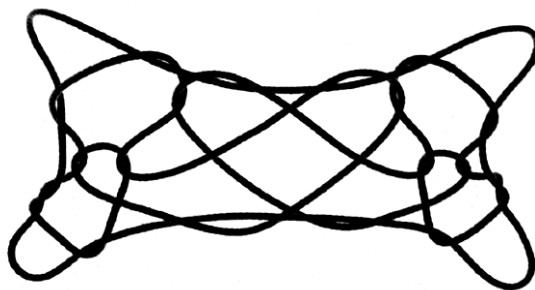


# Using String Figures to Teach Math Skills

## Part 6: Fun with Opening $\nabla$

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*One of several "heart-nets" created by the author*

### ABSTRACT

*In previous articles the author has explored various string figure "systems" suitable for teaching math students how to think logically and better comprehend various fundamental math concepts such as reciprocals, additive inverses, matrices, iterations, generalized formulas, chirality, asymmetry. By learning to create and symbolically notate new string figures students overcome their math shyness. In this playful article the author exhibits his acquired skills by showing how a new string figure opening can be systematically manipulated to create a series of beautiful new designs. Several net figures with a "heart" embedded in the web are of special interest.*

### INTRODUCTION

In the previous article my colleague Joe Ornstein introduced us to his non-traditional version of Opening A, which he calls Opening  $\nabla$  (Ornstein 2002). This missive is a paean to Joseph-of-the-Ornsteins for coming up with this marvelously intriguing new twist on an old standby. We all do different things

with the torus of string we play with. Opening  $\nabla$  offers us the opportunity to explore an entirely new framework of thought. The path of exploration i have chosen differs from that taken by Joe, and will probably differ from that taken by most other string figure enthusiasts who play with Joe's opening. We all contribute our unique perspectives.

## $\nabla$ NETS

My fondness for net-like string figures is well documented (Murphy 1997, 1998, 1999, 2000). i am also quite fond of string figures with "heart" motifs embedded in the design (Murphy 2001). The following exploration incorporates both. My prototypical ' $\nabla$ -Net' is made as follows:

### *Form the Loom*

- Opening  $\nabla$  (Insert each index and little finger, from below, into an untwisted loop; pass the right thumb away from you under the left near index string, then down behind the left palmar string and return to the right with the thumb still pointing down, carrying the left palmar string on its back; insert the left thumb, from the fingertip side, into the right thumb loop, then under the right near index string, then pass it down behind the right palmar string and return to the left through the right thumb loop with the left thumb still pointing down, carrying the right palmar string on its back; extend.)

### *First $\nabla$ -Net Weave*

- Pass each thumb away from you over the near index string, and with each thumb and index grasp the far index string and push it toward the center; with the left index and middle finger grasp the string held by the right thumb and index (release their grip), then with the right little finger reach over the right far index string and pick up the string held by the left thumb and index (release their grip); with the right thumb reach through the right little finger loop from above and pick up the string held by the left index and middle finger (release their grip), and return to give an upper right thumb loop. Finally, pass the left little finger over the left far index string and insert it, from above, into the upper right thumb loop, then withdraw the right thumb and return, thus transferring the upper loop to the left little finger. Extend with fingers pointing upward. You now have two loops on each little finger, a loop on each index, and a loop on each thumb. A palmar string passes through each index loop.
- Pass each index, from above, through the upper little finger loop; pick up the lower near little finger string and return through the upper loop.
- Drop both little finger loops and extend.
- Transfer the upper index loop to the little finger, inserting the little finger from below.

*Second  $\nabla$ -Net Weave*

- With each thumb pick up the near index string.
- Navajo the thumb loops (lift the lower loop over the upper loop and drop it the center of the figure).

The finishing phase i use for my  $\nabla$ -Net prototype is borrowed from the Klamath Net (Murphy 1999:169-172). First, shift the loops so that the two transverse strings are correctly positioned:

*Shifting the Loops (pre-Klamath)*

- Transfer the little finger loop to the index, inserting the index from above.
- Transfer the thumb loop to the three lesser fingers (i.e., insert the middle, ring, and little fingers into the thumb loop from below, close the near thumb string to the palm, and withdraw the thumb).
- Transfer the upper index loop to the thumb, inserting the thumb from below.

Now apply the *Klamath Net Ending* (see my 1999 article for illustrations):

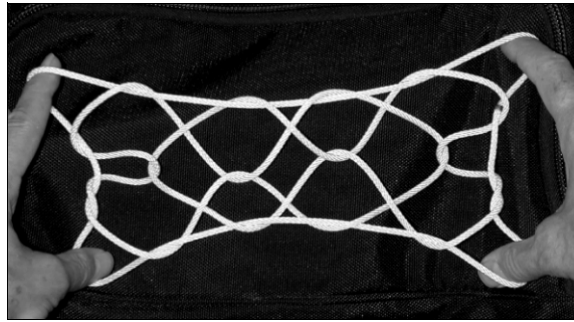
*Fixing the Bottom (Klamath)*

- Without withdrawing the middle finger from the lesser fingers loop, pass each middle finger toward you under the index loop and insert it, from above, into thumb loop; curl the middle finger around the far thumb string and draw it away from you through the lesser fingers loop; withdraw the ring-little fingers and close the middle finger to the palm; reinsert the ring-little fingers into the middle finger loop, closing the far middle finger string to the palm.
- Near each middle finger a loop surrounds the palmar string of each hand; the loop has an upper and a lower string (the latter being a transverse string); pass each middle finger toward you through this loop, then curl the middle finger around the lower string, drawing it away from you through the ring-little finger loop; drop the ring-little finger loop and reinsert these two fingers into the middle finger loop, closing the far middle finger string to the palm.

*Cleaning the Top (Klamath)*

- Withdraw the middle finger from the lesser fingers loop and insert it, from below, into the index loop; pinch the near thumb string between the tips of the index and middle fingers, and draw this string through the index-middle finger loop by rotating the index-middle finger pair away from you and up, thus placing the retrieved string on the back of each index (the index-middle finger loop slips off as you return).
- Drop the thumb loop. Withdraw the ring finger from its loop and extend the figure on index and little fingers only.

- *Optional:* If desired, transfer the little finger loop to the thumb, inserting the thumb from the fingertip side, and re-extend. The result is shown in fig. 1.



**Fig. 1** -  $\nabla$ -Net Prototype

A shorthand notation for this figure is:

Opening  $\nabla$ , First  $\nabla$ -Net Weave, Second  $\nabla$ -Net Weave, Shift Loops (pre-Klamath), Klamath Net Ending.

### ALTERING THE FINISHING PHASE

Rather than finishing with the Klamath Net Ending, try a *Modified Inuit Net Ending*. First, shift the loops so that the two transverse strings are correctly positioned:

#### *Shifting the Loops (pre-Inuit)*

- Transfer the little finger loop to the index, inserting the index from above.
- Transfer the thumb loop to the ring-little fingers (i.e., insert the ring and little fingers into the thumb loop from below, close the near thumb string to the palm, and withdraw the thumb).

Now proceed with the traditional *Inuit Net Ending* (see Murphy 1999:163-166 for illustrations) but pick up the *far* lower thumb string rather than the *near* lower thumb string when cleaning the top, as described below:

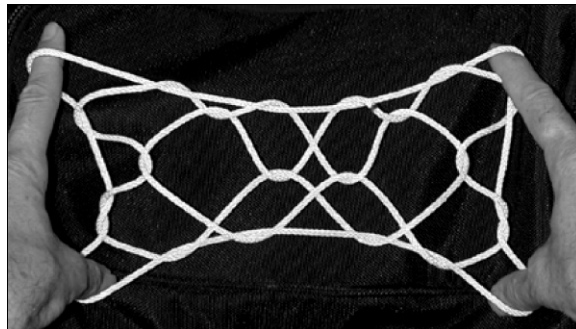
#### *Fixing the Bottom (Inuit)*

- Pass each thumb under both index loops and away from you through the ring-little finger loop, then pass the thumb up behind the strings and insert it, from below, into the upper index loop.
- With each thumb hook down the upper far index string and draw it down through the ring-little finger loop until the thumb is below the far little finger string, then with each thumb pick up the far little finger string and

- draw it through the loop on each thumb, which slips off as you return.
- Drop the ring-little finger loop and extend.

*Cleaning the Top (Inuit, modified)*

- Transfer the thumb loop to the little finger (i.e., insert the little finger into the thumb loop from below, close the near thumb string to the palm, and withdraw the thumb).
- Transfer both index loops to the thumb, maintaining their relative order.
- Insert each index, from above, into the upper thumb loop, then pick up the lower far thumb string and return through the upper loop.
- Insert middle finger, from below, into the index loop; pinch the upper near thumb string between the tips of the index and middle fingers, and draw this string through the index-middle finger loop by rotating the index-middle finger pair away from you and up, thus placing the retrieved string on the back of each index (the index-middle finger loop slips off as you return).
- Drop both thumb loops. Withdraw the ring finger from its loop and extend the figure on index and little fingers only.
- *Optional:* If desired, transfer the little finger loop to the thumb, inserting the thumb from the fingertip side, and re-extend. The result is shown in fig. 2.

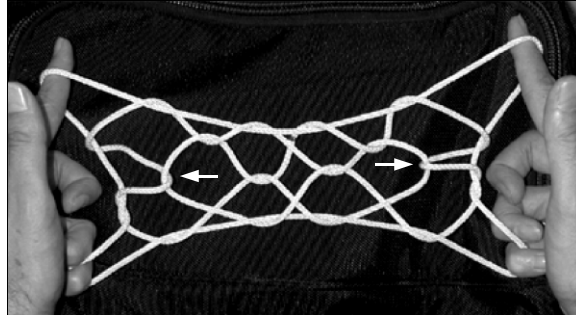


**Fig. 2 -  $\nabla$ -Net Prototype with Modified Inuit Ending**

A shorthand notation for this figure is:

Opening  $\nabla$ , First  $\nabla$ -Net Weave, Second  $\nabla$ -Net Weave, Shift Loops (pre-Inuit), Modified Inuit Net Ending.

When you compare this to the traditional Inuit Net (fig. 3), you can see differences in the wraps just above and inward from the outer double wraps (see arrows in fig. 3).



**Fig. 3 - Traditional Inuit Net**

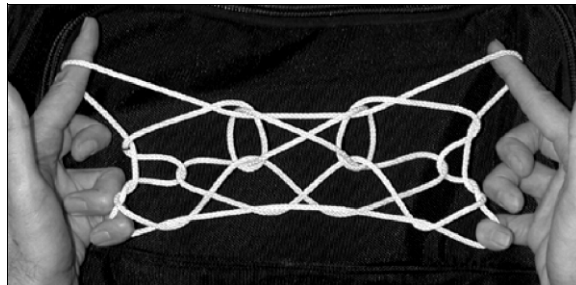
## ALTERING THE WEAVING PHASE

### LOOP PASSAGES (BRAIDING)

In my “Vertical Nets and Tennis Nets” article i introduced a method for doing a weave artificially by temporarily removing a loop from a finger, passing it up or down through another loop, then resetting the loop on its original finger (Murphy 2000:235-239). In the  $\nabla$ -Net sequence a loop passage is easily inserted between the Loom Phase and the Weaving Phase. First, do Opening  $\nabla$ , then proceed as follows:

- Pass your right thumb and index finger down through the left index loop; grasp the segment of string that crosses the back of the left little finger ; lift the left little finger loop off the left little finger, draw it up through the left index loop, then return the loop, without twisting it, to the left little finger. Repeat on the opposite hand.

Now do the standard First  $\nabla$ -Net Weave, but in the first paragraph replace the phrase “far index string” with the phrase “far thumb string”, then finish the figure as written. The resulting figure, shown in fig. 4, has two “eyes”.



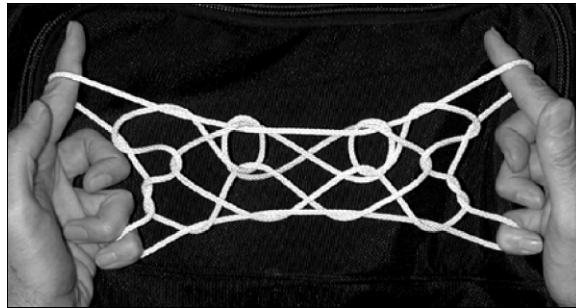
**Fig. 4 -  $\nabla$ -Net Variation with “Eyes”**

A shorthand notation for this figure is:

Opening  $\nabla$ , little finger loop up through index loop and return, First  $\nabla$ -Net Weave (with far thumb string), Second  $\nabla$ -Net Weave, Shift Loops (pre-Klamath), Klamath Net Ending (fig. 4).

As expected, weaving phase and finishing phase alterations can be combined to create new figures. In the above figure, if the Modified Inuit Net Ending is used instead of the Klamath Net Ending, a figure with better “eyes” results:

Opening  $\nabla$ , little finger loop up through index loop and return, First  $\nabla$ -Net Weave (with far thumb string), Second  $\nabla$ -Net Weave, Shift Loops (pre-Inuit), Modified Inuit Net Ending (fig. 5).



**Fig. 5 -  $\nabla$ -Net Variation with Better “Eyes”**

#### DOMINANT SWITCHES

In my “Vertical Nets and Tennis Nets” article i also introduced *Dominant Switches* (Murphy 2000:274-280) as a means of altering designs. A dominant switch is a loop exchange with fingers inserted from below rather than above during the transfers. In the  $\nabla$ -Net sequence a dominant switch of the thumb loops is easily introduced between the first and second weaves. Left and right versions are defined as follows:

##### *Left-Dominant Thumb Switch*

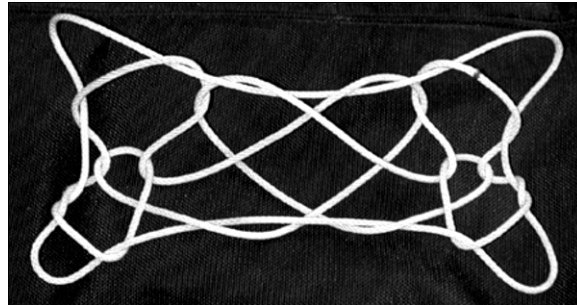
- Transfer the left thumb loop to the top of the right thumb, inserting the right thumb from below and from the near side; pass the left thumb down through the right upper thumb loop, then insert it, from below and from the near side, into the right lower thumb loop; lift this loop off the right thumb, drawing it up through the right upper thumb loop as you return the left thumb to its original position.

*Right Dominant Thumb Switch*

- Transfer the right thumb loop to the top of the left thumb, inserting the left thumb from below and from the near side; pass the right thumb down through the left upper thumb loop, then insert it, from below and from the near side, into the left lower thumb loop; lift this loop off the left thumb, drawing it up through the left upper thumb loop as you return the right thumb to its original position.

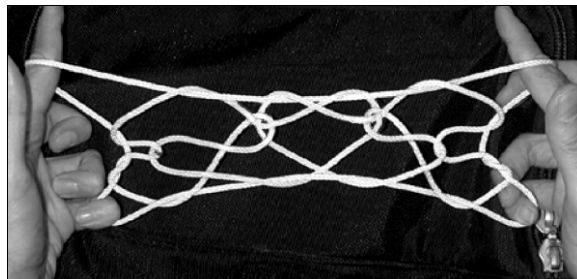
The following two figures have pleasing “heart” motifs embedded in the design. Both utilize a left dominant switch, but the endings differ.

Opening  $\nabla$ , First  $\nabla$ -Net Weave, Left-Dominant Thumb Switch, Second  $\nabla$ -Net Weave, Shift Loops (pre-Klamath), Klamath Net Ending (fig. 6).



**Fig. 6** -  $\nabla$ -Net with Embedded Heart (Klamath Net Ending)

Opening  $\nabla$ , First  $\nabla$ -Net Weave, Left-Dominant Thumb Switch, Second  $\nabla$ -Net Weave, Shift Loops (pre-Inuit), Modified Inuit Net Ending (fig. 7).

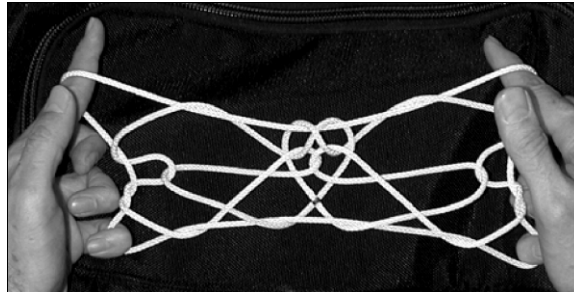


**Fig. 7** -  $\nabla$ -Net with Embedded Heart (Modified Inuit Net Ending)



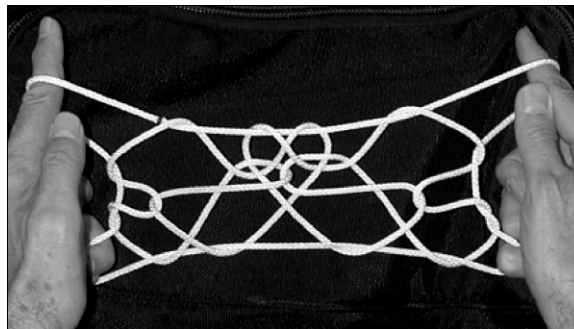
A right dominant switch introduces a “link” into the center of the heart. Compare the following two figures:

Opening  $\nabla$ , First  $\nabla$ -Net Weave, Right-Dominant Thumb Switch, Second  $\nabla$ -Net Weave, Shift Loops (pre-Klamath), Klamath Net Ending (fig. 8).



**Fig. 8** -  $\nabla$ -Net with Linked Heart (Klamath Net Ending)

Opening  $\nabla$ , First  $\nabla$ -Net Weave, Right-Dominant Thumb Switch, Second  $\nabla$ -Net Weave, Modified Inuit Net Ending (fig. 9).



**Fig. 9** -  $\nabla$ -Net with Linked Heart (Modified Inuit Net Ending)

A close-up view of each center is shown in figs. 10 and 11.



**Fig. 10** - Close-up View of Fig. 8



**Fig. 11** - Close-up View of Fig. 9

## ALTERING THE LOOM PHASE

### LOOP ROTATIONS

Loop rotations can be used to modify a loom as long as they do not alter the positions of the transversals, which ultimately become the frame lines of a net figure. Unlike Opening A, in which the transversals belong to the *thumb* and little finger loops, the transversals of Opening  $\nabla$  are belong to the *index* and little finger loops. Since half rotations of loops alter the positions of transversals, only the thumb loop of Opening  $\nabla$  can be rotated  $+\frac{1}{2}$  or  $-\frac{1}{2}$ :

*Note:* With Opening  $\nabla$ , the terms “from above” and “from below” are a bit ambiguous when describing the thumb loops since the thumbs point toward the maker or slightly down. But as always, “from above” should be interpreted as “from the fingertip side” and “from below” as “from the knuckle side”:

#### $+\frac{1}{2}$ thumb loop rotation

- Transfer the thumb loop to the index, inserting the index *from above*, then re-transfer this loop to the thumb, inserting the thumb *from below*.

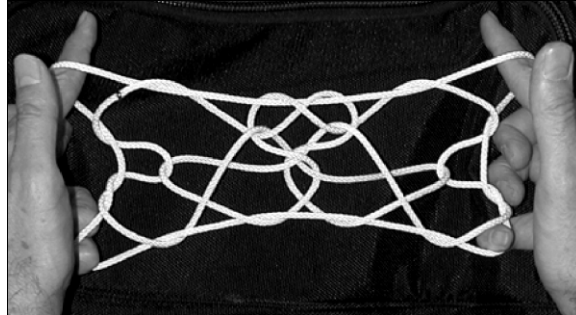
#### $-\frac{1}{2}$ thumb loop rotation

- Transfer the thumb loop to the index, inserting the index *from below*, then re-transfer this loop to the thumb, inserting the thumb *from above* (bend the index toward you and down, then complete the transfer).

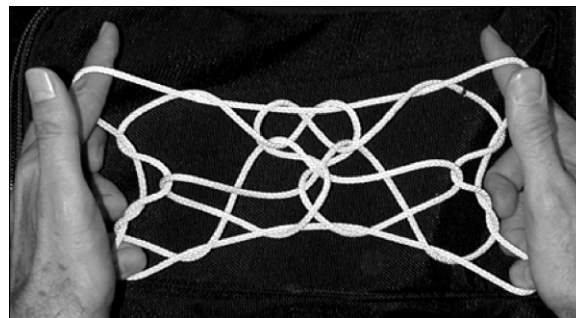
The following example incorporates a  $-\frac{1}{2}$  thumb loop rotation plus two tandem dominant switches.

Opening  $\nabla$ , thumb loop  $-\frac{1}{2}$ , left-dominant thumb switch, left-dominant thumb switch, First  $\nabla$ -Net Weave, Second  $\nabla$ -Net Weave, Shift Loops (pre-Klamath), Klamath Net Ending.

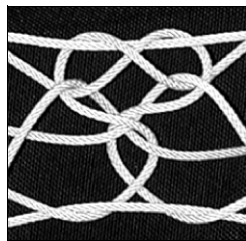
The result is shown in figs. 12 and 13 (front and back views). The center has an interesting complex crossing that resembles a collapsed trefoil (figs. 14 and 15), but the crossing tends to “pull” toward one side and appear asymmetrical when the figure is fully extended. However, when laid flat and arranged, the center is actually quite even and pretty.



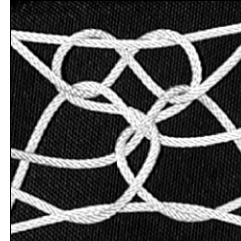
**Fig. 12** -  $\nabla$ -Net with Collapsed Trefoil (Front)



**Fig. 13** -  $\nabla$ -Net with Collapsed Trefoil (Back)



**Fig. 14** - Close-up View of Fig. 12

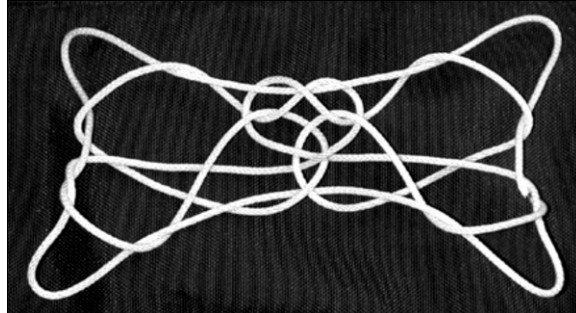


**Fig. 15** - Close-up View of Fig. 13

#### LOOM REPLACEMENTS

$\nabla$ -Nets do not necessarily require an Opening  $\nabla$  loom. Note that a design similar to fig. 12 can be made if a Left DNA Loom (Murphy 2001:213-214) is combined with a left-dominant index loop switch (Murphy 2000:274), and the Second  $\nabla$ -Net Weave is modified:

L-DNA, left-dominant index switch, First  $\nabla$ -Net Weave, thumb loop up through index loop and return, Shift Loops (pre-Klamath), Klamath Net Ending (fig. 16).

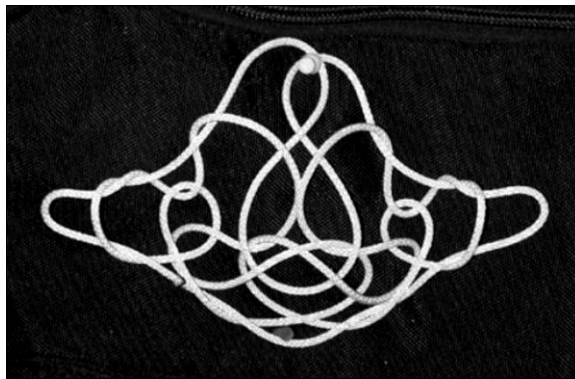


**Fig. 16 -  $\nabla$ -Net with Improved Trefoil**

The trefoil motif seen in fig. 16 is far better than the lop-sided one seen in fig. 12. Replacing Opening  $\nabla$  with an L-DNA opening requires replacing the Second  $\nabla$ -Net Weave with a loop passage maneuver because of differences in the initial positions of the transversals.

### ALTERING THE EXTENSION

As an epilogue i wish to remind the reader that string figure designs need not be displayed solely on the hands. Many rival ornamental knots when placed on the ground and arranged. If the index loops of the heart-net shown in fig. 6 are superimposed and pinned and the center of the lower frame line pulled down and likewise pinned, the pleasing design shown in fig. 17 is obtained.



**Fig. 17 - Fig. 6 with Index Loops Superimposed and Pinned**

i find it very interesting how pleasing bilateral symmetry is. The maskedness of the forms attained are probably due to the residual skill we inherited from our homonid ancestors of picking out eyes in the leafy canopy of the forest. If you take a picture of the leafy canopy and then arbitrarily alter the image with a computer along vertical lines so you distort it side-to-side in opposite directions you will tend to see faces in the images.

In closing, i offer the following poem:

little i, big I and seeing eye agree  
 and all nod their aye to me  
 there is no little, there is no big  
 the fund of sages all agree  
 there is no i, there is only me

big I is where the big bucks come  
 and not to poet casting eye  
 to words of ink in forming i

the I of big is what i'm not  
 the i of small is what i've got  
 the selfless self of i is heard  
 when i, I, and eye agree

murphy reefing his topsail in a blinding wind

*inoli*

### LITERATURE CITED

- Murphy, J. (1997) "Using String Figures to Teach Math Skills: Part 1—The Diamonds System." *Bulletin of the International String Figure Association* 4:56-74.
- Murphy, J. (1998) "Using String Figures to Teach Math Skills: Part 2— The Ten Men System." *Bulletin of the International String Figure Association* 5:159-209.
- Murphy, J. (1999) "Using String Figures to Teach Math Skills. Part 3— The North American Net System." *Bulletin of the International String Figure Association* 6:160-211.
- Murphy, J. (2000) "Using String Figures to Teach Math Skills. Part 4— Vertical Nets and Tennis Nets." *Bulletin of the International String Figure Association* 7:215-286.
- Murphy, J. (2001) "Using String Figures to Teach Math Skills. Part 5— Opening Theory." *Bulletin of the International String Figure Association* 8:211-234.
- Ornstein, J. (2002) "Opening  $\forall$ ." *Bulletin of the International String Figure Association* 9:294-306.